# Computer Aided Geometric Design

Curves

B-Spline Curve


Instructor: Dr. Prashant K. Jain

*Associate Professor ME Discipline*

PDPM Indian Institute of Information Technology, Design and Manufacturing Jabalpur, Jabalpur, INDIA

**Resources:** web.iiitdmj.ac.in/~pkjain/

Email: pkjain@iiitdmj.ac.in, pkjain2006@gmail.com

http://www.iiitdmj.ac.in/Faculty/pkjain.html

http://in.linkedin.com/in/pkjain2006

https://www.facebook.com/pkjain2006

PDPM

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, DESIGN AND MANUFACTURING JABALPUR

(An Institute of National Importance (INI) established by MHRD, Govt. of INDIA)


## Parametric Representation Of Free-form Curves

Interpolation Curves

- Parametric Cubic Curve

Approximation Curves

- Bezier Curve
- B-Spline Curve
- NURBS Curve



## B-Spline Curve

- Curve is defined by n+1 control points and the order ( k ) of the curve.
- The curve has an advantage that it has local propagation unlike global propagation properties of Bezier curve. i.e. Non global –each vertex $B_i$ is associated with unique basis.
- Each vertex affects the shape only over a range of parameter value where its associated basis function is nonzero.
- B spline basis also allows the order of basis function and hence the degree of resulting curve can be changed without changing the number of vertices.
- The curve is made up of (n-k+2) segments.
- Only k control points affect any segment of the curve.



## B-Spline Curve

B spline curve is given by

$$p(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t) \qquad t_{min} < t \le t_{max}, \quad 2 < k \le n+1$$

$B_i$- Position vector of n+1 defining vertices

$N_{i,k}$- Normalized B-Spline basis function

For i[th] normlized B-Spline basis function of order k (degree k-1) the basis function $N_{i,k}(t)$ defined as: (cox de-boor formula)

$$N_{i,1}(t) = \begin{cases} 1, & if\ x_i \le t \le x_{i+1} \\ 0, & otherwise \end{cases}$$

and

$$N_{i,k}(t) = \frac{(t - x_i) N_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t) N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$

$X_i$ are elements of knot vector where $x_i \le x_{i+1}$

The function p(t) is polynomial of degree k-1 on each interval



## B-Spline Curve

**Input to B-Spline Curve**

1st point, $p_0 = (x_0, y_0, z_0)$

2nd point, $p_1 = (x_1, y_1, z_1)$

|

nth point, $p_n = (x_n, y_n, z_n)$

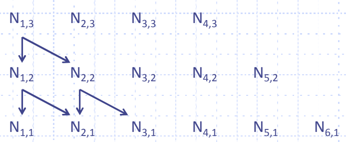
Order of curve = k

Knot vector

Equation for B-Spline curve of four defining vertices and order k=3 will be:

$$p(t) = N_{1,3}B_1 + N_{2,3}B_2 + N_{3,3}B_3 + N_{4,3}B_4$$

The basis function dependencies for $N_{i,3}$ are as:

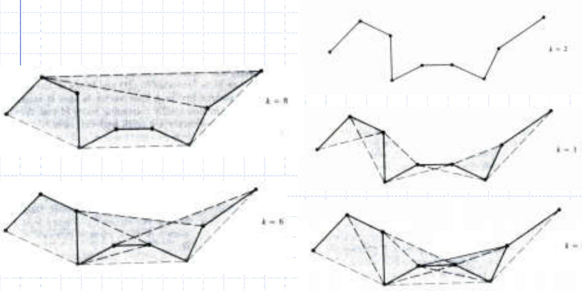



## B-Spline Curve

- Each basis function is positive or zero for all parameter values, i.e., $N_{i,k} \ge 0$
- The sum of the rational B-spline basis for any parameter value t is one, i.e., $\sum_{i=1}^{n+1} N_{i,k}(t) = 1$
- Except for k=1, each basis has precisely one maximum.
- B-spline curve of order k (degree k-1) is $c^{k-2}$ continuous everywhere.
- The maximum order of B-spline curve is equal to the number of defining polygon vertices.
- B-spline curve exhibits the variation diminishing property.
- B-spline curve generally follows the shape of the defining polygon.
- P(t) and its derivatives of order 1,2,-------k-2, are all contains over the entire cure.

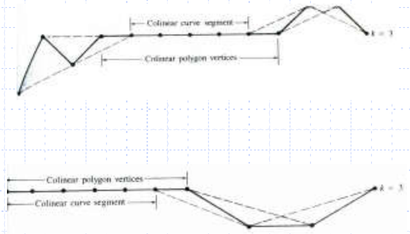
## B-Spline Curve

- B-spline curve lies within the union of convex hulls formed by k successive defining polygon vertices.
- *Convex hull property of B-spline curves is stronger than Bezier Curves.*





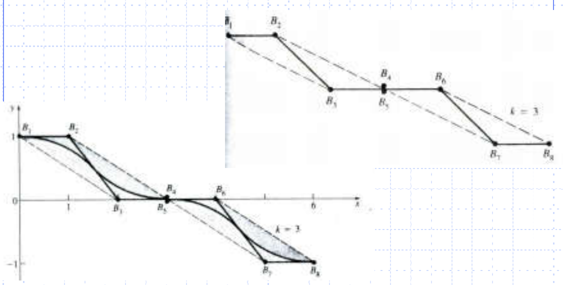
## Linear Segment in B-Spline Curve






## B-Spline Curve

If at least k-1 coincident defining polygon vertices occur i.e .,
$B_i = B_{i+1}$ …………………$B_{i+k-2}$ then convex hull of $B_i$ to $B_{i+k-2}$ is the vertex itself,
Hence the resulting B- spline curve must pass through the vertex Bi.





## B-Spline Curve: Knot Vectors

Fundamentally three types of knot vector are used:
**Uniform, Open uniform, Non-uniform**

In a **uniform knot vector**, knot values are uniformly / evenly spaced

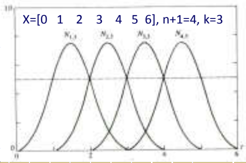
[0 1 2 3 4]
[-0.2 -0.1 0 0.1 0.2 ]

In practice uniform knot vectors begin at zero are incremented by 1 to some maximum value or normalized in the range between 0 to 1. i.e.,

[0 0.25 0.5 0.75 1.0]

For a given order *k* uniform knot vector yield periodic uniform basis function for which

$$N_{i,k}(t) = N_{i-1,k}(t-1) = N_{i+1,k}(t-1)$$




*Each basis function is a translate of the other*



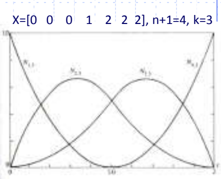
## B-Spline Curve: Knot Vectors

An **open uniform knot vector** has multiplicity of knot values at the ends equal to the order K of B-Spline function.
Internal knot values are evenly spaced, e.g.

k=2 [0 0 1 2 3 4 4]
k=3 [0 0 0 1 2 3 3 3]
k=4 [0 0 0 0 1 2 2 2 2]

or for normalized increments

k=2 [0 0 ¼ ½ ¾ 1 1]
k=3 [0 0 0 ⅓ ⅔ 1 1 1]
k=4 [0 0 0 0 ½ 1 1 1 1]




Formally an open uniform knot vector is given by

$x_i = 0$ $1 \le i \le k$
$x_i = i-k$ $k+1 \le i \le n+1$
$x_i = n-k+2$ $n+2 \le i \le n+k+1$

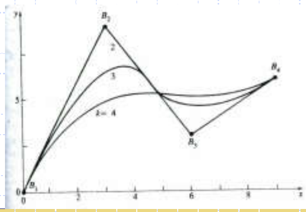


## B-Spline Curve: Knot Vectors

The resulting **open uniform basis function** yields a curve that behave nearly like Bezier curves.

When number of defining polygon vertices is equal to the order of the B-Spline basis and a open uniform knot vector is used the B-Spline basis is reduces to the Bernstein basis, hence the resulting B spline curve is a Bezier curve.

In that case knot vector [0 0 0 0 1 1 1 1] with four polygon vertices results in a cubic Bezier Curve.

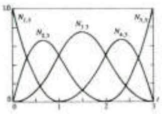


## B-Spline Curve: Knot Vectors

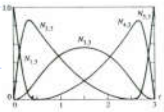
**Non uniform knot vectors** may have either unequally spaced and/or multiple internal knot values, they may be periodic or open. e. g.

[0 0 0 1 1 2 2 2]
[0 1 2 2 3 4]
[0 0.28 0.5 0.72 1]

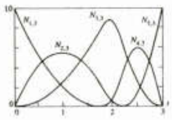
*Basis function for n+1 = 5, k = 3*



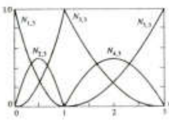
[x] = [0 0 0 1 2 3 3 3]



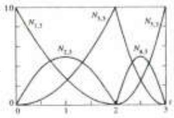
[x] = [0 0 0 0.4 2.6 3 3 3]



[x] = [0 0 0 1.8 2.2 3 3 3]



[x] = [0 0 0 1 1 3 3 3]



[x] = [0 0 0 2 2 3 3 3]



## Calculating Periodic Basis Function

Calculate four third order (k=3) basis function

$$N_{(i,3)}(t),\ i = 1,2,3,4$$

Here n+1, the number of basis function is 4, and curve is

$$p(t) = N_{1,3}B_1 + N_{2,3}B_2 + N_{3,3}B_3 + N_{4,3}B_4$$

The basis function dependencies for $N_{i,3}$ are as:

$N_{1,3}$ $N_{2,3}$ $N_{3,3}$ $N_{4,3}$

$N_{1,2}$ $N_{2,2}$ $N_{3,2}$ $N_{4,2}$ $N_{5,2}$

$N_{1,1}$ $N_{2,1}$ $N_{3,1}$ $N_{4,1}$ $N_{5,1}$ $N_{6,1}$



## Calculating Periodic Basis Function

$$N_{(i,1)}(t), i = \begin{cases} 1, if\ x_i \le t < x_{i+1} \\ 0, otherwise \end{cases}$$

Now knot vector range needed:

$$N_{i,k}(t) = \frac{(t - x_i)N_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t)N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$

Equation shows that calculation of $N_{6,1}$ requires knot values $X_6$ and $X_7$, while calculation of $N_{1,1}$ requires $x_1$ and $x_2$. Thus knot values form 0 to n+k are required.

Number of knot values is thus n+k+1. Hence the knot vector:

$$[x] = [0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$$

Where $x_1$=0 , $x_2$=1 - - - - -$x_7$=6 and Parameter and range is $0 \le t \le 6$.



## Calculating Periodic Basis Function

The basis function for various parameter ranges are:

$0 \le t < 1$

$$N_{1,1}(t) = 1, \quad N_{i,1}(t) = 0, \quad i \ne 1$$
$$N_{1,2}(t) = t, \quad N_{i,2}(t) = 0, \quad i \ne 1$$
$$N_{1,3}(t) = \frac{t^2}{2}, \quad N_{i,3}(t) = 0, \quad i \ne 1$$

$1 \le t < 2$

$$N_{2,1}(t) = 1, \quad N_{i,1}(t) = 0, \quad i \ne 2$$
$$N_{1,2}(t) = (2 - t), \quad N_{2,2}(t) = (t - 1) \quad N_{i,2}(t) = 0, \quad i \ne 1,2$$
$$N_{1,3}(t) = \frac{t}{2}(2 - t) + \left(\frac{3 - t}{2}\right)(t\text{-}1),$$
$$N_{2,3}(t) = \frac{(t-1)^2}{2}, \quad N_{i,3}(t) = 0, \quad i \ne 1,2,3$$



## Calculating Periodic Basis Function

$2 \le t < 3$

$$N_{3,1}(t) = 1, \quad N_{i,1}(t) = 0, \quad i \ne 3$$
$$N_{2,2}(t) = (3 - t), \quad N_{3,2}(t) = (t - 2), \quad N_{i2}(t) = 0, \quad i \ne 2,3$$
$$N_{1,3}(t) = \frac{(3-t)^2}{2}; \quad N_{2,3}(t) = \frac{(t-1)(3-t)}{2} + \frac{(4-t)(t-2)}{2};$$
$$N_{3,3}(t) = \frac{(t-2)^2}{2}, \quad N_{i3}(t) = 0, \quad i \ne 1,2,3$$

$3 \le t < 4$

$$N_{4,1}(t) = 1, \quad N_{i,1}(t) = 0, \quad i \ne 4$$
$$N_{3,2}(t) = (4 - t), \quad N_{4,2}(t) = (t - 3), \quad N_{i2}(t) = 0, \quad i \ne 3,4$$
$$N_{2,3}(t) = \frac{(4-t)^2}{2}; \quad N_{3,3}(t) = \frac{(t-2)(4-t)}{2} + \frac{(5-t)(t-3)}{2};$$
$$N_{4,3}(t) = \frac{(t-3)^2}{2}, \quad N_{i3}(t) = 0, \quad i \ne 2,3,4$$



## Calculating Periodic Basis Function

$4 \le t < 5$

$$N_{5,1}(t) = 1, \quad N_{i,1}(t) = 0, \quad i \ne 5$$
$$N_{4,2}(t) = (5 - t), \quad N_{5,2}(t) = (t - 4), \quad N_{i2}(t) = 0, \quad i \ne 4,5;$$
$$N_{3,3}(t) = \frac{(5-t)^2}{2};$$
$$N_{4,3}(t) = \frac{(t-3)(5-t)}{2} + \frac{(6-t)(t-4)}{2};$$
$$N_{i3}(t) = 0; \quad i \ne 3,4$$

$5 \le t < 6$

$$N_{6,1}(t) = 1, \quad N_{i,1}(t) = 0, \quad i \ne 6$$
$$N_{5,2}(t) = (6 - t), \quad N_{i2}(t) = 0, \quad i \ne 5;$$
$$N_{4,3}(t) = \frac{(6-t)^2}{2}, \quad N_{i3}(t) = 0, \quad i \ne 4$$

## Periodic Basis Functions

$N_{1,1}(t)=1, \quad 0 \le t < 1$

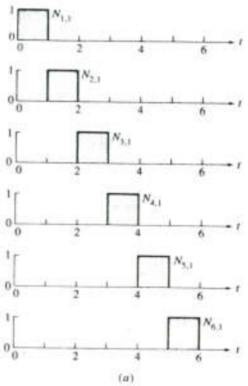
$N_{2,1}(t)=1, \quad 1 \le t < 2$

$N_{3,1}(t)=1, \quad 2 \le t < 3$

$N_{4,1}(t)=1, \quad 3 \le t < 4$

$N_{5,1}(t)=1, \quad 4 \le t < 5$

$N_{6,1}(t)=1, \quad 5 \le t < 6$





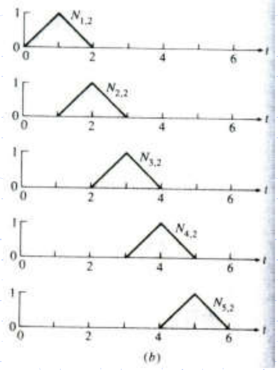
## Periodic Basis Functions

$N_{1,2}(t)=t, \quad 0 \le t < 1$

$N_{1,2}(t)=(2-t), \quad 1 \le t < 2$

$N_{2,2}(t)=(t-1) \quad 1 \le t < 2$

$N_{2,2}(t)=(3-t), \quad 2 \le t < 3$



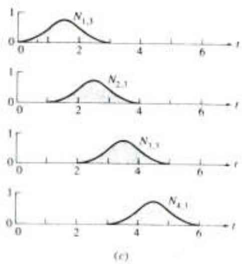


## Periodic Basis Functions

$N_{1,3}(t)=\frac{t^2}{2}, \quad 0 \le t < 1$

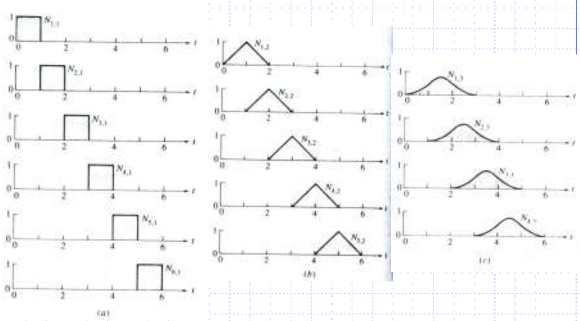
$N_{1,3}(t)=\frac{t}{2}(2-t)+\left(\frac{3-t}{2}\right)(t-1), \quad 1 \le t < 2$

$N_{1,3}(t)=\frac{(3-t)^2}{2}, \quad 2 \le t < 3$





## Periodic Basis Functions





## Calculating Open Uniform Basis Function

Calculate for (n=3) third order basis function (k=3) basis function $N_{i,3}$ i= 1,2,3,4. with an open knot vector.

Open knot vector with integer intervals between internal knot values is

$x_i = 0 \qquad 1 \le i \le k$

$x_i = i-k \qquad k+1 = 1 \le i \le n+1$

$x_i = n-k+2 \qquad n+2 \le i \le n+k+1$

Parameter range is $0 \le t \le n-k+2$ , i.e., zero to maximum knot value and number of knot values n+k+1

Therefore the knot vector is:

**[x] = [0 0 0 1 2 2 2]**

Where $x_1=0$ , $x_2=0$ - - - - $-x_7=2$ and Parameter and range is $0 \le t \le 2$.



## Calculating Open Uniform Basis Function

The basis function for various parameter ranges are:

**$0 \le t < 1$**

$N_{3,1}(t) = 1 \quad N_{i,1}(t)=0 \quad i \ne 3$

$N_{2,2}(t) = 1-t \quad N_{3,2}(t)=t \quad N_{i,2}(t)=0 \quad i \ne 2,3$

$N_{1,3}(t) = (1-t)^2 \quad N_{2,3}(t) = t(1-t)+(2-t)t/2$

$N_{3,3}(t) = t^2/2 \quad N_{i,3}(t) = 0 \quad i \ne 1,2,3$

**$1 \le t < 2$**

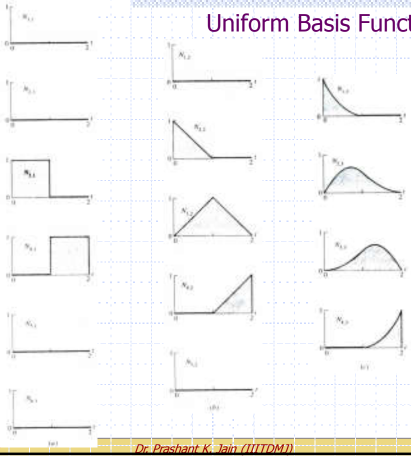
$N_{4,1}(t) = 1 \quad N_{i,1}(t) = 0 \quad i \ne 4$

$N_{3,2}(t) = (2-t) \quad N_{4,2}(t) = (t-1) \quad N_{i,2}(t) = 0 \quad i \ne 3,4$

$N_{2,3}(t) = (2-t)^2/2 \quad N_{3,3}(t) = t(2-t)/2+(2-t)(t-1)$

$N_{4,3}(t) = (t-1)^2 \quad N_{i,3}(t) = 0 \quad i \ne 2,3,4$

## Uniform Basis Functions





## Calculating Non-uniform Basis Function

Calculate the five (n+1 = 5 ) third – order (k = 3) basis functions $N_{i,3}$ (t) , i=1, 2,3, 4, 5 using the knot vector [X] = [0 0 0 1 1 3 3 3] which contains an interior repeated knot value.

$$N_{i,1}(t) = \begin{cases} 1, \text{ if } x_i \le t \le x_{i+1} \\ 0, \text{ otherwise} \end{cases}$$

and

$$N_{i,k}(t) = \frac{(t - x_i) N_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t) N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$

**0 ≤ t < 1**

$N_{3,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 3$

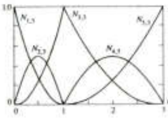
$N_{2,2}(t) = 1-t$ ; $N_{3,2}(t) = t$ ; $N_{i,2}(t) = 0$, $i \neq 2,3$

$N_{1,3}(t) = (1-t)^2$ ; $N_{2,3}(t) = t(1-t) + (1-t)t = 2t(1-t)$;

$N_{3,3}(t) = t^2$ ; $N_{i,3}(t) = 0$, $i \neq 1,2,3$



## Calculating Non-uniform Basis Function




[x] = [0 0 0 1 1 3 3 3]

**1 ≤ t < 1**

$N_{i,1}(t) = 0$, all i

$N_{i,2}(t) = 0$, all i

$N_{i,3}(t) = 0$, all i

Notice specifically that as a consequence of the multiple knot value, $N_{4,1}(t) = 0$ for all t.

**1 ≤ t < 3**

$N_{5,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 5$

$N_{4,2}(t) = (3-t)/2$ ; $N_{5,2}(t) = (t-1)/2$ ; $N_{i,2}(t) = 0$, $i \neq 4,5$

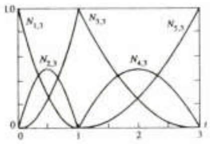
$N_{3,3}(t) = (3-t)^2/4$ ;

$N_{4,3}(t) = (t-1)(3-t)/4 + (3-t)(t-1)/4 = (3-t)(t-1)/2$ ;

$N_{5,3}(t) = (t-1)^2/4$ ; $N_{i,3}(t) = 0$, $i \neq 3, 4,5$



## Calculating Non-uniform Basis Function




[x] = [0 0 0 1 1 3 3 3]

Notice that for each value of t the $\sum N_{i,k}(t) = 1.0$ . For example, with

**0 ≤ t < 1**

$$\sum_{i=1}^{5} N_{i,3}(t) = (1-t)^2 + 2t(1-t) + t^2$$

$= 1-2t+t^2+2t-2t^2+t^2 = 1$

Similarly, for **1≤ t < 3**

$$\sum_{i=1}^{5} N_{i,3}(t) = \frac{1}{4}[(3-t)^2 + 2(3-t)(t-1) + (t-1)^2]$$

$= ¼[9-6t+t^2-6t+8t-2t^2+1-2t+t^2] = 4/4 = 1$



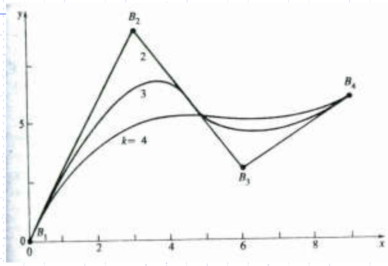
## Flexibility in B-Spline Curves

The shape of B-spline curve can be controlled by :

- Changing type of knot vector and hence basis function
- Changing order (k) of the basis function
- Using multiple polygon vertices
- Using multiple knot value in the knot vector
- Changing number and position of defining polygon vertices



## Effect of varying order on open B-spline curve



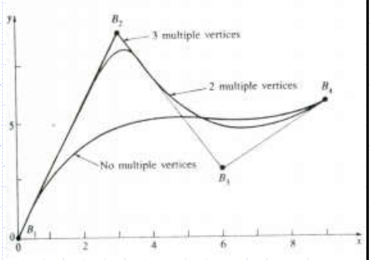
## Effect of multiple or coincident vertices

Multiple vertices at $B_2$, k=4

For k=4
1= [0 0 0 0 1 1 1 1]
[$B_1$ $B_2$ $B_3$ $B_4$ ]

2= [0 0 0 0 1 2 2 2 2]
[$B_1$ $B_2$ $B_2$ $B_3$ $B_4$]

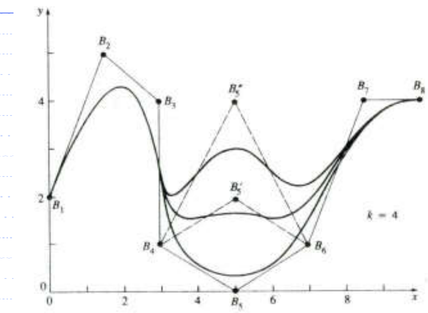
3= [0 0 0 0 1 2 3 3 3 3]
[$B_1$ $B_2$ $B_2$ $B_2$ $B_3$ $B_4$]






## Local control of B-Spline Curves

Changing position of defining polygon vertices at $B_5$






## Derivatives of B spline curve at any point

$$p(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t)$$

$$p'(t) = \sum_{i=1}^{n+1} B_i N'_{i,k}(t)$$

$$N'_{i,k}(t) = \frac{N_{i,k-1}(t) + (t - x_i) N'_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t) N'_{i+1,k-1}(t) - N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$

$$N'_{i,1}(t) = 0$$

for all t and for k=2

$$N'_{i,2}(t) = \frac{N_{i,1}(t)}{x_{i+k-1} - x_i} - \frac{N'_{i+1,1}(t)}{x_{i+k} - x_{i+1}}$$



## Derivatives of B spline curve at any point

$$p''(t) = \sum_{i=1}^{n+1} B_i N''_{i,k}(t)$$

$$N''_{i,k}(t) = \frac{2N_{i,k-1}(t) + (t - x_i) N''_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t) N''_{i+1,k-1}(t) - 2N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$

$N''_{i,1}(t) = 0$ and $N''_{i,2}(t) = 0$ For all t.

For k=3

$$N''_{i,3}(t) = 2\left(\frac{N'_{i,2}(t)}{x_{i+k-1} - x_i} - \frac{N'_{i+1,2}(t)}{x_{i+k} - x_{i+1}}\right)$$



## Numericals: Calculating an open B-Spline Curve

Given $B_1$ [1 1] , $B_2$ [2,3], $B_3$[4,3] and $B_4$[3,1] the vertices of a Bezier polygon. Calculate both second and fourth – order B – Spline curves.

For k= 2 the open knot vector is [0 0 1 2 3 3]

Where $x_1$ =0 , $x_2$ = 0, ……, $x_6$ =3. The parameter range is 0≤ t ≤3.
The curve is composed of three linear (k-1 =1) segments.
For 0≤ t ≤3 the basis functions are :

**0 ≤ t < 1**
$N_{2,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 2$
$N_{1,2}(t) = 1-t$ ; $N_{2,2}(t) = t$ ; $N_{i,2}(t) = 0$, $i \neq 1,2$

**1 ≤ t < 2**
$N_{3,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 2$
$N_{2,2}(t) = 2-t$ ; $N_{3,2}(t) = (t-1)$ ; $N_{i,2}(t) = 0$, $i \neq 2,3$

**2 ≤ t < 3**
$N_{4,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 4$
$N_{3,2}(t) = (3-t)$ ; $N_{4,2}(t) = (t-2)$ ; $N_{i,2}(t) = 0$, $i \neq 3,4$



## Numericals: Calculating an open B-Spline Curve

Using equation $p(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t)$ $\quad t_{min} < t \leq t_{max}$ $\quad 2 < k \leq n+1$

The parametric B-Spline curve is

$P(t) = B_1 N_{1,2}(t) + B_2 N_{2,2}(t) + B_3 N_{3,2}(t) + B_4 N_{4,2}(t)$

For each of these intervals the curve is given by

$P(t) = (1-t)B_1 + tB_2 = B_1 + (B_2 - B_1)t$ $\quad 0 \leq t < 1$
$P(t) = (2-t)B_2 + (t-1)B_3 = B_2 + (B_3 - B_2)t$ $\quad 1 \leq t < 2$
$P(t) = (3-t)B_3 + (t-2)B_4 = B_3 + (B_4 - B_3)t$ $\quad 2 \leq t < 3$

In each case the result is the equation of the parametric straight line for the polygon span, i.e., the 'curve' is the defining polygon.

## Numericals: Calculating an open B-Spline Curve

The last point on the curve ($t = t_{max} = 3$) requires special consideration. Because of the open right – hand interval in equation

$$N_{(i,1)}(t), i = \begin{cases} 1, \text{if } x_i \le t < x_{i+1} \\ 0, \text{otherwise} \end{cases}$$

All the basis function $N_{i,K}$ at t = 3 are zero. consequently, the last polygon point does not technically lie on the B-Spline curve. However, practically it does.

Consider t = 3-Ɛ where Ɛ is an infinitesimal value.

letting Ɛ→0 shows that in the limit the last point on the curve and the last polygon point are coincident.

Practically, this result is incorporated by either artificially adding the last polygon point to the curve description or by defining N ($t = t_{max}$) = 1.0 .



## Numericals: Calculating an open B-Spline Curve

For k=4 the order of the curve is equal to the number of defining polygon vertices.

Thus the B-spline curve reduces to a Bezier curve. The knot vector with $t_{max}$ = n-k+2 = 3-4+2 =1 is [0 0 0 0 1 1 1 1].

The basis functions are:

**0 ≤ t < 1**

$N_{4,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 4$

$N_{3,2}(t) = (1-t)$ ; $N_{4,2}(t) = t$ ; $N_{i,2}(t) = 0$, $i \neq 3,4$

$N_{2,3}(t) = (1-t)^2$ ; $N_{3,3}(t) = 2t(1-t)$ ;

$N_{4,3}(t) = t^2$; $N_{i,3}(t) = 0$, $I \neq 2,3,4$

$N_{1,4}(t) = (1-t)^3$ ; $N_{2,4}(t) = t(1-t)^2 + 2t(1-t)^2 = 3t(1-t)^2$;

$N_{3,4}(t) = 2t^2(1-t) + (1-t)t^2 = 3t^2(1-t)$; $N_{4,4}(t) = t^3$;



## Numericals: Calculating an open B-Spline Curve

Using equation $p(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t) \quad t_{\min} < t \le t_{\max} \quad 2 < k \le n+1$

The parametric B-Spline curve is

$P(t) = B_1 N_{1,4}(t) + B_2 N_{2,4}(t) + B_3 N_{3,3}(t) + B_4 N_{4,4}(t)$

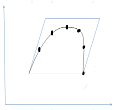
$P(t) = (1-t)^3 B_1 + 3t(1-t)^2 B_2 + 3t^2(1-t) B_3 + t^3 B_4$

Thus, at t=0 $P(0) = B_1$

and at t= ½ $P(\frac{1}{2}) = \frac{1}{8}B_1 + \frac{3}{8}B_2 + \frac{3}{8}B_3 + \frac{1}{8}B_4$

and $P(\frac{1}{2}) = \frac{1}{8}[1 \ 1] + \frac{3}{8}[2 \ 3] + \frac{3}{8}[4 \ 3] + \frac{1}{8}[3 \ 1]$

= [ 11/4 5/2]





THANK YOU