Computer Aided Geometric Design

Curves

Re-parameterization
Truncating, extending, subdividing of curves
Composite Curves, Continuity conditions

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Reparameterization

- Change in parametric interval so that neither the shape nor position of the curve change

  v = f(u)

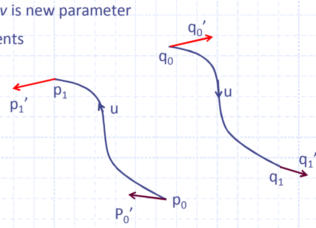
  for changing the direction of curve

  v = -u where *v* is new parameter

  B matrix of geometric coefficients

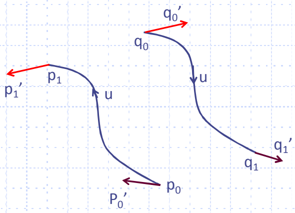
  $B_1 = [p_0 \;\; p_1 \;\; p_0' \;\; p_1']^T$

  $B_2 = [q_0 \;\; q_1 \;\; q_0' \;\; q_1']^T$



Reparameterization

$$\begin{bmatrix} p_0 \\ p_1 \\ p_0' \\ p_1' \end{bmatrix} \Rightarrow \begin{bmatrix} q_0 \\ q_1 \\ q_0' \\ q_1' \end{bmatrix}$$



If

$q_0 = p_1$

$q_1 = p_0$

$q_0' = -p_1'$

$q_1' = -p_0'$

Then two curves are identical except that they have opposite directions of parameterization

Then $B_2 = [P_1 \;\; P_0 \;\; -P_1' \;\; -P_0']$

Reparameterization

- For a more general case curve is initially parameterized from u_i to u_j and want to change parametric variable ranges from v_i to v_j.
- Let $B_1 = [p_i \;\; p_j \;\; p_i' \;\; p_j']^T$ and $B_2 = [q_i \;\; q_j \;\; q_i' \;\; q_j']^T$
- The end points are invariant or insensitive to any change of parameterization, so $q_i = p_i$ and $q_j = p_j$ to maintain constant position.
- Tangent vectors are sensitive to the functional relationship between u and v, i.e. v = f(u), a linear relationship is required to preserve cubic form.

  Thus v = au + b then dv = a du

  also $v_i = au_i + b$ and $v_j = au_j + b$ from this a and b can be found and relationship between tangent vectors

$$q' = \frac{u_j - u_i}{v_j - v_i} p'$$

Reparameterization

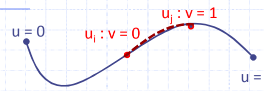
Now complete relationship between two sets of geometric coefficients

$$q_i = p_i$$
$$q_j = p_j$$
$$q_i' = \frac{u_j - u_i}{v_j - v_i} p_i'$$
$$q_j' = \frac{u_j - u_i}{v_j - v_i} p_j'$$

- This also shows that tangent vector magnitudes must change to accommodate a change in the range of the parametric variable.
- Magnitudes are simply scaled by the ratio of the ranges of parametric variable.
- This preserves the direction of tangent vectors and shape of the curve.

Truncating, Extending and Subdividing

- Find new B matrix for a truncated or extended pc curve



- curve truncated at u_i and u_j from segment u_0 to u_i and from u_j to u_i. we can represent remaining curve segment as a full pc curve from v_0 to v_1.
- Compute p_i and p_j using p = UMB and p_i' and p_j' using p' = U'MB.
- Ratio of parametric interval length $(u_j - u_i)/(v_j - v_i)$ reduces to $u_j - u_i$, since $v_j - v_i = 1$. Then

$$q_0 = p_i \qquad q_1 = p_j$$
$$q_0' = (u_j - u_i) p_0' \qquad q_1' = (u_j - u_i) p_1'$$

- To subdivide a pc curve into *n* successive segments of arbitrary length and generate n new pc curves, then geometric coefficient or B matrix of i[th] segment are

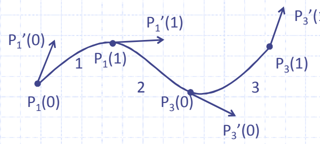
$$B_i = \begin{bmatrix} p_{i-1} & p_i & (u_i - u_{i-1})p'_{i-1} & (u_i - u_{i-1})p'_i \end{bmatrix}$$

- If a curve is divided into n equal segments, then

$$B_i = \begin{bmatrix} p_{(i-1)/n} & p_{i/n} & \frac{1}{n}p'_{(i-1)/n} & \frac{1}{n}p'_{i/n} \end{bmatrix}$$

Composite Curves

- When two or more curves segments are joined together, they form a continuous composite curve



- Blending a new curve between two existing curve
- Let $B_1 = [P_1(0)\ \ P_1(1)\ \ P_1'(0)\ \ P_1'(1)]$
- and $B_3 = [P_3(0)\ \ P_3(1)\ \ P_3'(0)\ \ P_3'(1)]$
- End point must coincide
- So that $P_2(0) = P_1(1)$ and $P_2(1) = P_3(0)$

- Further, the tangent at each end of the new curve must match the tangent of adjoining curve.
- Magnitude may be different.
- Thus

$$P_2'(0) = a\frac{P_1'(1)}{|P_1'(1)|}$$

and

$$P_2'(1) = b\frac{P_3'(0)}{|P_3'(0)|}$$

Then
$$B_2 = \begin{bmatrix} P_1(1) & P_3(0) & a\frac{P_1'(0)}{|P_1'(0)|} & b\frac{P_3'(0)}{|P_3'(0)|} \end{bmatrix}$$

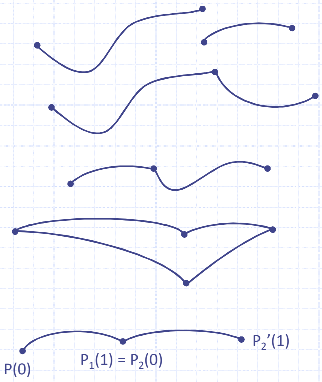
Where a and b are positive scale factor

- Expression for any PC curve smoothly and continuously blended with preceding and succeeding curves:

$$B_i = \begin{bmatrix} P_{i-1}(1) & P_{i+1}(0) & a\frac{P_{i-1}'(1)}{|P_{i-1}'(1)|} & b\times\frac{P_{i+1}'(0)}{|P_{i+1}'(0)|} \end{bmatrix}$$

Continuity Conditions

- C^0 continuity
  - No gaps

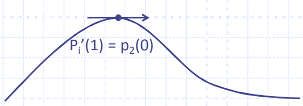


- C^1 continuity between two curves required a common tangent line at their joining point.
- C^2 continuity also requires that two curves posses equal curvature at their joint.

$p_i(1) = p_{i+1}(0)$

$p_i'(1) = K_1\, p_{i+1}'(0)$

$p_i''(1) = K_2\, p_{i+1}''(0)$



Que.: Given a cubic hermite curve whose geometric coefficients are

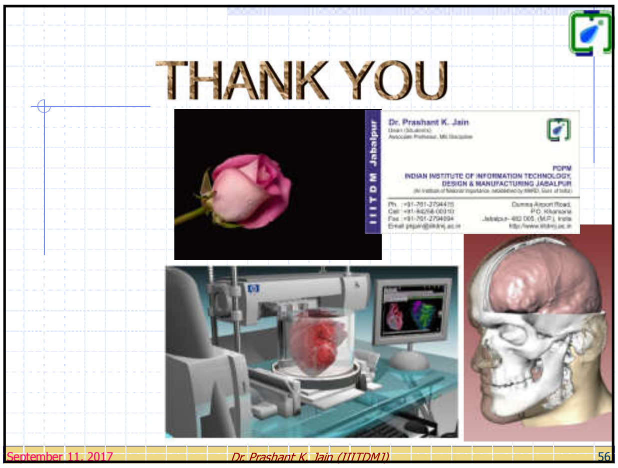
$$B = \begin{bmatrix} P_0 & P_1 & P_0' & P_1' \end{bmatrix}$$

truncate the curve at u = 0.2 and u = 0.7 and reparameterize the remaining segment so that v ϵ [0 1]. Find the relationship between the geometric coefficients of truncated curve and those of original.

Que.: Given a cubic hermite curve whose geometric coefficients are

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 4 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

sub divide this curve into three segments with joints at u = 1/3 and u = 2/3. Reparameterize each segment over a unit interval and compute the three sets of geometric coefficients.


THANK YOU
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56